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Generalization of Quasi Principally Injective S-Acts

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Abstract

The purpose of this paper is to introduce a new kind of generalization of quasi principally injective S-acts over monoids (QP-injective), (and hence generalized quasi injective), namely quasi small principally injective S-acts. Several properties of this kind of generalization are discussed. Some of these properties are analogous to that notion of quasi small principally injective for general modules. Characterizations of quasi small principally injective acts are considered. Conditions are investigated under which subacts are inheriting quasi small principally injective property.

Keywords- Quasi Small Principally Injective S-Acts, M-Cyclic Small Subacts , Fully Invariant Small Subacts , Projective Acts, Quasi Principally Injective Acts

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I. INTRODUCTION

In this paper we extended concept of the previous works and to generalize new concepts which are: to extend the concept of principally injective acts, to generalize the concept of quasi principally injective acts[1], to establish and extend some new concepts which are dual to quasi principally injective acts [14] and quasi small principally injective acts. Also, we are interested in seeing extend the characterizations and properties of acts remain valid for these previous concepts.

In everywhere of this paper, every S-acts is unitary right S-acts with zero element Θ which denoted by M_s . We refer the reader to the references ([1],[2],[3],[4],[5],[6],[7],[9],[10],[11],[15]) for basic definitions and terminology relating to S-acts over monoid and injective(projective) acts which are used here.

In [1], the author introduced the concept of quasi principally injective S-act as a generalization of quasi injective. An S-acts N_s is called M-principally injective if for every S-homomorphism from M-cyclic subact of M_s into N_s can be extended to an S-homomorphism from M_s into N_s (if this is the case , we write N_s is M-P-injective) . An S-act M_s is called quasi-principally injective if it is M-P-injective, that is every S-homomorphism from M-cyclic subact of M_s to M_s can be extended to S-endomorphism of M_s (for simply QP-injective) .

Recently, we adopt the concept of small quasi principally injective S-acts over monoids which represents, on one hand, a generalization of quasi principally injective S-acts and on the other hand representing a generalization of quasi-small principally injective modules [16]. We study their characterizations and properties. Some results on quasi principally injective S-acts [1] and [14] extended to these S-acts.

II. QUASI SMALL PRINCIPALLY INJECTIVE S-ACTS

A. Definition (2.2)

Let M_s be a right S-acts. A right S-acts N_s is called M-small principally injective (for short MSP-injective) if, every S-homomorphism from an M-cyclic small sub-acts of M_s to N_s can be extended to an S-homomorphism from M_s to N_s. Equivalently, for any endomorphism α of M_s with α(M) is small in M_s , every S-homomorphism from α(M) to N_s can be extended to an S-homomorphism from M_s to N_s .

A right S-acts M_s is called quasi small principally injective (for short quasi SP-injective) if it is MSP-injective.

B. Remark and Example (2.2)

1) Let S = $\begin{pmatrix} F & F \\ 0 & F \end{pmatrix}$, where F is a field with T= End (M_s) , and M_s = $\begin{pmatrix} F & F \\ 0 & 0 \end{pmatrix}$. Then M_s is MSP-injective S-acts.

1) Proof

It is easy to show that A = $\begin{pmatrix} 0 & F \\ 0 & 0 \end{pmatrix}$ is the only nonzero small M-cyclic sub-acts of M_s . Let $\alpha \in T$ such that $\alpha\left(\begin{pmatrix} F & F \\ 0 & 0 \end{pmatrix}\right) \subseteq \begin{pmatrix} 0 & F \\ 0 & 0 \end{pmatrix}$. Since $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \in M_s$, $\alpha\left(\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\right) = \begin{pmatrix} 0 & x \\ 0 & 0 \end{pmatrix}$ for some $0 \neq x \in F$. Then for each $y,z \in F$

$\alpha\left(\begin{pmatrix} y & z \\ 0 & 0 \end{pmatrix}\right) = \alpha\left[\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} y & z \\ 0 & 0 \end{pmatrix}\right] = \alpha\left(\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\right)\begin{pmatrix} y & z \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & x \\ 0 & 0 \end{pmatrix}\begin{pmatrix} y & z \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$. It implies that $\alpha = 0$. It implies that $\ell_T(\ker(\alpha)) = 0 = T\alpha$. This means that M_s is MSP-injective S-sets.

2) If N is small sub-acts of an S-acts M_s , then $\dot{\bigcup}_{i=1}^{n} N_i$ is small sub-acts of M_s .

2) Proof

The proof will be by induction on n. For n = 1 , the assertion holds by the assumption . Assume that $N = N_1 \dot{\cup} N_2 \dot{\cup} \ldots \dot{\cup} N_{n-1}$ is small sub-acts of M_s . Now, for sub-acts B of M_s , we have $N \dot{\cup} N_n \dot{\cup} B = M_s$. As N is small, so $N_n \dot{\cup} B = M_s$ and so $B = M_s$ since N_n is small sub-acts of M_s .Thus, the proof is complete.

3) The Z-act with multiplication Z is small-injective but not injective.

C. Proposition (2.3)

Let M_s and N_i ($1 \leq i \leq n$) be a right S-acts. Then, $\bigoplus_{i=1}^{n} N_i$ is MSP-injective if and only if N_i is MSP-injective for each i = 1,2,..,n.

1) Proof

The necessity is clear. If for the sufficiency we prove the result when n = 2 , then it is enough . Let $\alpha \in T$ with $\alpha(M)$ is small in M_s and $f : \alpha(M) \longrightarrow N_1 \oplus N_2$ be an S-homomorphism. Since N_1 (N_2) are MSP-injective , then there exists S-homomorphism $f_1: M_s \longrightarrow N_1$ ($f_2: M_s \longrightarrow N_2$) such that $f_1 i = \pi_1 f$ ($f_2 i = \pi_2 f$) where $\pi_1(\pi_2)$ is the projection map of from $N_1 \oplus N_2$ into $N_1(N_2)$ and $i: \alpha(M) \longrightarrow M_s$ is the inclusion map . Put $j_1 f_1 = \bar{f}$($j_2 f_2 = \bar{f}$) Thus $\bar{f}$ extends f.

Next corollary represents a generalization of lemma (2.3.11) (1) in [12]

D. Corollary (2.4)

Retract of an MSP- injective S-acts is also MSP-injective.

The following theorem is a generalization of theorem (2.3) in [16]:

E. Theorem (2.5)

The following conditions are equivalent for projective S-acts M_s:

1) Every M-cyclic small sub-act of M_s is projective.
2) Every factor of an MSP-injective S-act is MSP-injective.
3) Every factor of an injective S-act is MSP-injective.

1) Proof

($1 \longrightarrow 2$) Let A_s be an MSP-injective S-acts and $\sigma(M)$ be M-small sub-acts in M_s. Let $\alpha: \sigma(M) \longrightarrow A_s/\rho$ be S-homomorphism . Then by (1), there exists S-homomorphism $\beta: \sigma(M) \longrightarrow A_s$ such that $\pi\beta = \alpha$ where $\pi: A_s \longrightarrow A_s/\rho$ is the natural epimorphism. Since A_s is MSP-injective, so β can be extended to S-homomorphism $f: M_s \longrightarrow A_s$ .Put $\varphi = \pi f$, so φ is the extension of α to M_s .

($2 \longrightarrow 3$) Assume that E is injective S-acts and E/ρ is the factor of E. Since every injective is MSP-injective acts, so E is MSP-injective acts. Then, by (2) E/ρ is MSP-injective acts.

($3 \longrightarrow 1$) Let $\alpha(M)$ be M-cyclic small sub-acts of M_s and $f: A_s \longrightarrow B_s$ be an S-epimorphism, where A_s and B_s be two S-acts. Then $B_s \cong A_s/\rho$, where the congruence $\rho = \ker(f)$. Let $g: \alpha(M) \longrightarrow B_s$. Since every S-acts can be embedding in injective acts by corollary (1.6) [8, p.186] , so embed A_s in injective acts E. Then $B_s \cong A_s/\rho$ is a sub-acts of E/ρ, so by (3) g is extends to $\bar{g}: M_s \longrightarrow E/\rho$. As M_s is projective, so $\bar{g}$ can be lifted to $\sigma: M_s \longrightarrow E$. It is obvious that $\sigma(\alpha(M)) \subset A_s$. Put $\sigma i = \beta$, where i is the inclusion map of $\alpha(M)$ into M_s . This means that $\beta: \alpha(M) \longrightarrow A_s$. Thus g lifted to β .

For the endomorphism monoid , we have the following proposition :

F. Proposition (2.6)

Let M_s be a right S-acts and T=End (M_s) . Then, the following conditions are equivalent:

1) M_s is quasi small principally-injective.
2) $\ell_T(\ker(\alpha)) = T\alpha$ for all $\alpha \in T$ with $\alpha(M_s)$ small in M_s.
3) $\ker(\alpha) \subseteq \ker(\beta)$, where $\alpha,\beta \in T$ with $\alpha(M_s)$ small in M_s , implies $T\beta \subseteq T\alpha$.
4) $\ell_T(\ker(\alpha) \cap (\beta(M) \times \beta(M))) = \ell_T(\beta(M) \times \beta(M)) \cup T\alpha$ for $\alpha, \beta \in T$ with $\alpha(M_s)$ small in M_s .
5) If $\sigma: \alpha(M) \longrightarrow M_s$, $\alpha \in T$ with $\alpha(M_s)$ small in M_s , then $\sigma\alpha \in T\alpha$.

1) Proof

($1 \longrightarrow 2$) Let $\alpha \in T$ with $\alpha(M)$ is small in M_s and let $\beta \in \ell_T(\ker(\alpha))$. Then $\ker(\alpha) \subseteq \ker(\beta)$, so there exists an S-homomorphism $\sigma: \alpha(M) \longrightarrow N_s$ such that $\sigma\alpha = \beta$. Since $\alpha(M_s)$ is small in M_s and M_s is quasi small P-injective , so there exists an S-homomorphism

$\overline{\sigma}: M_s \longrightarrow M_s$ such that $\overline{\sigma}i = \sigma$, where $i : \alpha(M) \longrightarrow M_s$ is the inclusion map . Therefore $\beta = \overline{\sigma}\alpha \in T\alpha$. For the other direction let $\beta \in T\alpha$, then $\beta = \sigma\alpha$ for some $\sigma \in T$. For each s,t ∈ S with ms=mt , we have $\alpha(ms) = \alpha(mt)$ and then $\sigma\alpha(ms) = \sigma\alpha(mt)$. Thus, it is implies that $\beta(ms) = \beta(mt)$, and so $\beta \in \ell_T(\ker(\alpha))$.
(2→1) Let $\alpha \in T$ with $\alpha(M)$ is small in M_s and $\sigma : \alpha(M) \to M_s$ be an S-homomorphism . Then $\sigma\alpha \in T$ and $\sigma\alpha \in \ell_T(\ker(\alpha))$. By assumption, $\sigma\alpha = f\alpha$ for some $f \in T$. This implies that M_s is quasi small P-injective.
(2→3) Let $\ker(\alpha) \subseteq \ker(\beta)$, where α , $\beta \in T$ with $\alpha(M)$ is small in M_s . Then , $\ell_T(\ker(\beta)) \subseteq \ell_T(\ker(\alpha))$. Since $T\beta \subseteq \ell_T(\ker(\beta))$ and by(2) we have $\ell_T(\ker(\alpha)) = T\alpha$, so $T\beta \subseteq T\alpha$.
(3→4) Let $\sigma \in \ell_T(\ker(\alpha) \cap (\beta(M) \times \beta(M)))$. We claim that $\ker\alpha\beta \subseteq \ker\sigma\beta$, for this let $(m_1,m_2) \in \ker\alpha\beta$, so $\alpha\beta(m_1) = \alpha\beta(m_2)$. This implies that $(\beta(m_1), \beta(m_2) \in (\ker(\alpha) \cap (\beta(M) \times \beta(M)))$. Then $\sigma\beta(m_1) = \sigma\beta(m_2)$. Thus $(m_1,m_2) \in \ker\sigma\beta$. By (3) , we have $T\sigma\beta \subseteq T\alpha\beta$ and $\sigma\beta = u\alpha\beta$ for some $u \in T$. This means that there is $u \in T$ such that $\sigma\beta = u\alpha\beta$ for each $\alpha,\beta \in T$. In particular $\sigma = u\alpha$. Thus $\sigma \in \ell_T(\beta(M) \times \beta(M)) \cup T\alpha$. Conversely, let $\sigma \in \ell_T(\beta(M) \times \beta(M)) \cup T\alpha$, so this means that $\sigma \in \ell_T(\beta(M) \times \beta(M))$ or $\sigma \in T\alpha$ (this means that $\sigma = u\alpha$ for some $u \in T$) . If $\sigma \in \ell_T(\beta(M) \times \beta(M))$, then this means that $\sigma\beta(m_1) = \sigma\beta(m_2)$ for each $m_1,m_2 \in M_s$. Now, for each $m_1,m_2 \in M_s$, we have $(\beta m_1, \beta m_2) \in (\ker(\alpha) \cap (\beta(M) \times \beta(M)))$. If $\sigma = u\alpha$, then $\alpha\beta(m_1) = \alpha\beta(m_2)$ and hence $u\alpha\beta(m_1) = u\alpha\beta(m_2)$. Thus $\sigma\beta(m_1) = \sigma\beta(m_2)$ and then $\sigma \in \ell_T(\ker(\alpha) \cap (\beta(M) \times \beta(M)))$.
(4→5) Put $\beta = I_M$, identity map of M_s , then we have $\sigma\alpha \in \ell_T(\ker(\alpha)) = \ell_T(\ker(\alpha) \cap (\beta(I_M) \times \beta(I_M)) = \ell_T(\beta(I_M) \times \beta(I_M)) \cup T\alpha = T\alpha$.
(5→1) It is obvious.

G. Corollary (2.7)

The following conditions are equivalent for monoid S:

1) S is SP-injective.
2) $\ell_S(\gamma_S(a)) = Sa$ for all $a \in S$ with aS is small in S_s
3) $\gamma_S(b) \subseteq \gamma_S(a)$, where a,b ∈ T with aS is small in S_s , implies $Sa \subseteq Sb$.
4) $\ell_S(bS \cap (\gamma_S(a) \times \gamma_S(a)) = \ell_S(b \times b) \cup Sa$ for a,b ∈ S with aS is small in S_s .
5) If $\sigma: aS \longrightarrow S_s$, $a \in S$ with aS is small in S_s , then $\sigma a \in Sa$.

H. Proposition (2.8)

Retract of a quasi-small principally injective S-act is quasi- small principally injective.

1) Proof

Let M_s be quasi small principally injective S-acts and N is a retract of M_s . Let $\alpha \in T$=End (N) with $\alpha(N)$ be N-cyclic small sub-acts of N and then in M_s (this means that $\alpha(N)$ is M-cyclic small sub-acts of M_s) by lemma (3.4) in [13] and let $f : \alpha(N) \to N$ be S-homomorphism . Since M_s is quasi small principally injective S-acts, so there exists $g: M_s \to M_s$ such that $g i_N i_{\alpha(N)} = j_N f$ where j_N is the injection of N into M_s . Let $\overline{g}(= g_{|N}): N \to N$. Thus, it is clear that $\overline{g}$ is extension of f and N is quasi small principally injective S-acts.

I. Proposition (2.9)

Let M_s be quasi SP-injective S-acts and $\alpha_i \in T$ with $\alpha_i(M)$ is M-cyclic small sub-acts of M_s ($1 \leq i \leq n$)

1) If $T\alpha_1 \oplus T\alpha_2 \oplus \ldots \oplus T\alpha_n$ is direct , then any S-homomorphism $\beta : \alpha_1(M) \dot{\cup}\, \alpha_2(M)\, \dot{\cup} \ldots \dot{\cup}\, \alpha_n(M) \longrightarrow M_s$ has an extension in T .
2) If $\alpha_1(M) \oplus \alpha_2(M) \oplus \ldots \oplus \alpha_n(M)$ is direct, then $T(\alpha_1, \alpha_2, \ldots, \alpha_n) = T\alpha_1 \dot{\cup} T\alpha_2 \dot{\cup} \ldots \dot{\cup} T\alpha_n$.

1) Proof

Let $\beta : \alpha_1(M) \dot{\cup}\, \alpha_2(M)\, \dot{\cup} \ldots \dot{\cup}\, \alpha_n(M) \longrightarrow M_s$ be an S-homomorphism with $\alpha_i(M)$ is M-cyclic small sub-acts of M_s . Since M_s is quasi SP-injective S-acts, so there exists an S-homomorphism $\sigma_i : M_s \longrightarrow M_s$ such that $\sigma_i\alpha_i(m) = \beta\alpha_i(m)$ for all $m \in M_s$. Since $\alpha_1(M) \dot{\cup}\, \alpha_2(M)\, \dot{\cup} \ldots \dot{\cup}\, \alpha_n(M)$ is small sub-acts of M_s by remark and example(2.2)(2) , so β can be extended to $\overline{\beta}: M_s \longrightarrow M_s$ such that for any $m \in M_s$, $\overline{\beta}(\dot{\cup}_{i=1}^{n}\alpha_i)(m) = \beta(\dot{\cup}_{i=1}^{n}\alpha_i)(m)$. This implies that $\dot{\cup}_{i=1}^{n}\overline{\beta}\alpha_i = \dot{\cup}_{i=1}^{n}\sigma_i\alpha_i$. Since $T\alpha_1 \oplus T\alpha_2 \oplus \ldots \oplus T\alpha_n$ is direct, so $\overline{\beta}\alpha_i = \sigma_i\alpha_i$ for all ($1 \leq i \leq n$). Thus, $\overline{\beta}$ is an extension of β .

Let $\beta_1\alpha_1 \dot{\cup} \beta_2\alpha_2 \dot{\cup} \ldots \dot{\cup} \beta_n\alpha_n \in T\alpha_1 \dot{\cup} T\alpha_2 \dot{\cup} \ldots \dot{\cup} T\alpha_n$. Define $\sigma_i:(\alpha_1,\alpha_2,\ldots,\alpha_n)(M) \longrightarrow M_s$,where $\alpha_i \in T$ for each i by $\sigma_i((\alpha_1, \alpha_2, \ldots, \alpha_n)(m)) = \alpha_i(m)$ for every $m \in M_s$. Since $\alpha_1(M) \oplus \alpha_2(M) \oplus \ldots \oplus \alpha_n(M)$ is direct, so σ_i is well-defined. For this let $(\alpha_1, \alpha_2, \ldots, \alpha_n)(m) = (\beta_1, \beta_2, \ldots, \beta_n)(m)$ for each α_i , $\beta_i \in T$ and $m \in M_s$, this implies that $(\alpha_1 m, \alpha_2 m, \ldots, \alpha_n m) = (\beta_1 m, \beta_2 m, \ldots, \beta_n m)$, then $\alpha_i(m) = \beta_i(m)$. Thus $\sigma_i((\alpha_1, \alpha_2, \ldots, \alpha_n)(m)) = \sigma_i((\beta_1, \beta_2, \ldots, \beta_n)(m))$. As ($\dot{\cup}_{i=1}^{n}\alpha_i)(M)$ is small sub-acts of M_s by remark and example(2.2)(2) , and since M_s is quasi SP-injective S-acts , so there exists an S-homomorphism $\overline{\sigma}_i \in T$ which is extension of σ_i . Then $\alpha_i = \sigma_i(\alpha_1, \alpha_2, \ldots, \alpha_n) = \overline{\sigma}_i(\alpha_1, \alpha_2, \ldots, \alpha_n) \in T(\alpha_1, \alpha_2, \ldots, \alpha_n)$. This implies that $T\alpha_1 \dot{\cup} T\alpha_2 \dot{\cup} \ldots \dot{\cup} T\alpha_n \subseteq T(\alpha_1, \alpha_2, \ldots, \alpha_n)$. The reverse inclusion is always holds.

J. Proposition (2.10)

Let M_s be quasi SP-injective S-acts with $T = End(M_s)$, and let A be small sub-act of M_s . Let $\bigoplus_{i=1}^{n} \alpha_i(M)$ be direct sum of small M-cyclic sub-act of M_s . Then for any small A of M_s , we have : $A \cap \bigoplus_{i=1}^{n} \alpha_i(M) = \bigoplus_{i=1}^{n} (A \cap \alpha_i(M))$.

1) Proof

Let $x \in \bigoplus_{i=1}^{n} (A \cap \alpha_i(M))$, then there exists $j \in I=\{1,2,\ldots n\}$, such that $x \in A \cap \alpha_j(M)$ which implies that $x \in A$ and $x \in \alpha_j(M)$ for some $j \in I$, so $x \in A \cap \bigoplus_{i=1}^{n} \alpha_i(M)$. Then $\bigoplus_{i=1}^{n} (A \cap \alpha_i(M)) \subseteq A \cap \bigoplus_{i=1}^{n} \alpha_i(M)$. Conversely, let $a \in A \cap \bigoplus_{i=1}^{n} \alpha_i(M)$ which implies that $a \in A$ and $a \in \bigoplus_{i=1}^{n} \alpha_i(M)$. So there exists $j \in I$ such that $a \in \alpha_i(M)$. Let $\pi_j : \bigoplus_{i=1}^{n} \alpha_i(M) \longrightarrow \alpha_j(M)$ be the projection , then take $\sigma(= \pi_{j\,|\,\alpha_j(M)}) : \alpha_j(M) \longrightarrow \alpha_j(M)$. Let i_1, i_2 be the inclusion maps of $\alpha_i(M)$ and $\alpha_j(M)$ into M_s respectively . Since $\alpha_i(M)$ (where $i = \{1,2,\ldots,n\}$) is small sub-acts of M_s and M_s is quasi SP-injective acts , so by(1) of proposition(2.9) , σ can be extended to S-homomorphism $\beta : M_s \longrightarrow M_s$ (that is there exists $\beta \in T$, so β extends π_j . Thus for $a \in \alpha_j(M)$, we have $\alpha_j(m_j) = \pi_j(a) = \beta(a) = \sigma(a)$. Then, $a \in \bigoplus_{i=1}^{n} (A \cap \alpha_i(M))$ and $A \cap \bigoplus_{i=1}^{n} \alpha_i(M) \subseteq \bigoplus_{i=1}^{n} (A \cap \alpha_i(M))$.

III. CONCLUSIONS

From this work, we can put a highlight on some important points:
First: We have illustrated in the theorem (2.3) when the direct sum of finite M-small principally injective is also M-small principally injective and in corollary (2.4), clarified when the direct summand (retract) of M-small principally injective, is also M-small principally injective
Second: In theorem (2.5), we found the relationship between the factor of injective and M-small principally injective acts under projective condition. Besides, we found when M-cyclic subact of projective is projective?
Third: Proposition (2.6), corollary (2.7), and proposition (2.9) demonstrated the relationship between endomorphism monoid and acts under M-small principally injective property.
For the future work, one can extend this work by taking subacts as small finitely generated.

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