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Dynamics in an Undamped Series RLC Circuit

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Abstract

A two dimensional model of Series RLC circuit is considered for discussion. The system possess only the trivial equilibrium point and local stability conditions are obtained. The phase portraits are obtained for suitable parameter values. Oscillations and damping effects are illustrated with numerical examples.

Keywords- LCR Circuit, Differential Equations, Oscillation, Numerical Simulation, Stability

I. INTRODUCTION

In 1800, Alessandro Volta's invention of the battery which could produce a continuous flow of current made possible the development of the first electric circuit [2, 3, 5]. It was in 1890s where the RLC circuit was used in spark-gap radio transmitters to allow the receiver to be tuned to the transmitter. The Decay of oscillations in a RLC circuit which is used in Filters and oscillators are increased by introducing Resistor to the circuit. The theory of stability is an important branch of qualitative theory of Differential equations. Recently, [1] investigated the Hyers-Ulam stability and Hyers-Ulam-Rassias stability of a second order linear homogeneous and non-homogeneous differential equation of a LCR Electric Circuit Model. We here discuss the dynamics of the RLC circuit and provide the numerical simulations for linear homogenous differential equations of RLC circuit without damping.

The Second order Differential Equation in terms of charge q for a series RLC circuit is given by [4]

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q(t) = F(t) \qquad (1)$$

Where R is the effective Resistance, L is the Inductance of the Inductor component, C is the Capacitance of the capacitor components and F(t) is the Forcing function.

Equation (1) takes the form

$$\frac{d^2q}{dt^2} + 2\alpha\frac{dq}{dt} + \omega_0^2 q(t) = G \qquad (2)$$

Where the damping coefficient $\frac{R}{2L}$ is denoted by α , the resonant frequency $\frac{1}{\sqrt{LC}}$ denoted by ω_0 and the forcing term $\frac{F(t)}{L}$ by G.

II. MODEL DISCUSSION

A. System of Differential Equations

Equation (2) in the absence of the forcing term and the damping term takes the form

$$\frac{dq}{dt} = x(t), \quad \frac{dx}{dt} = -\omega_0^2 q(t) \qquad (3)$$

Where q(t) is the charge across the capacitor. Linearization process is useful technique in the study of system of differential equations. This involves the determination of equilibrium points and the evaluations of eigen values of the corresponding Jacobian Matrix. The system (3) has the trivial equilibrium point (0,0). The Jacobian matrix for the system (3) is

$$J(q,x) = \begin{pmatrix} 0 & 1 \\ -\omega_0^2 & 0 \end{pmatrix}$$

From the Jacobian matrix, the eigenvalues are $\lambda_{1,2} = \pm i\,\omega_0^2$. Thus, the eigenvalues are purely imaginary.

The Oscillation of the system (3) in the absence of damping term depends on the resonant frequency ω_0.Though the System remains stable for theresonant frequency $\omega_0 > 0$, the attainment of the equilibrium position by the system varies as ω_0 increases.Figure-1 exhibits the oscillation of the system (3)when $0 < \omega_0 < 1$. The time taken to attain stability is too large for the smaller values of ω_0.

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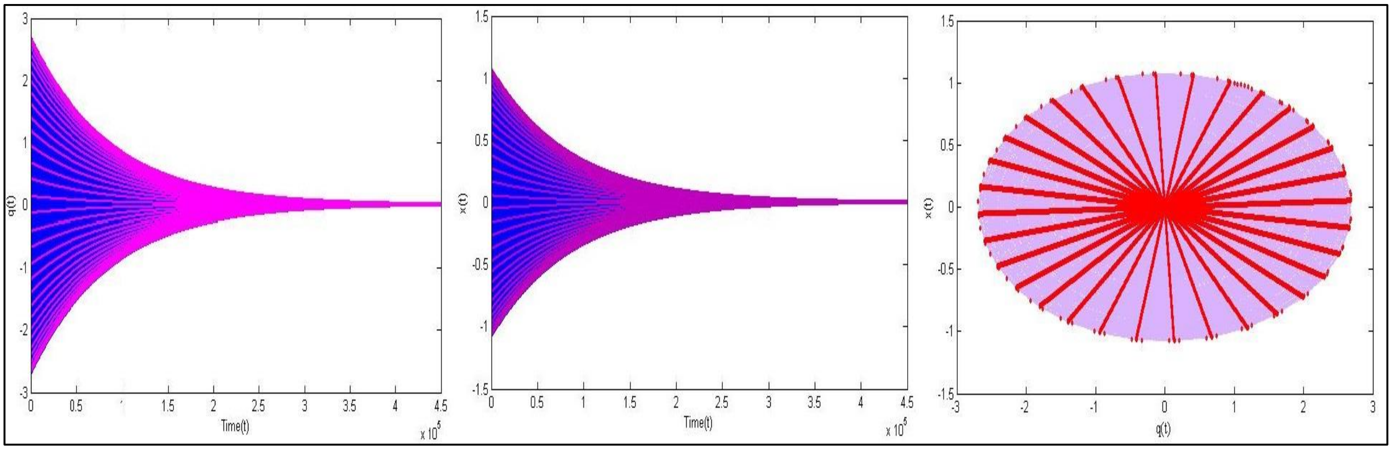

Fig. 1: Un Damped motion of (3) for $\omega_0 = 0.4$

Figure-2presents the time trajectory and phase portrait of the system (3) for $1 < \omega_0 \leq 2$. Clearly, the system (3) reaches stability more quickly compared to the previous case.

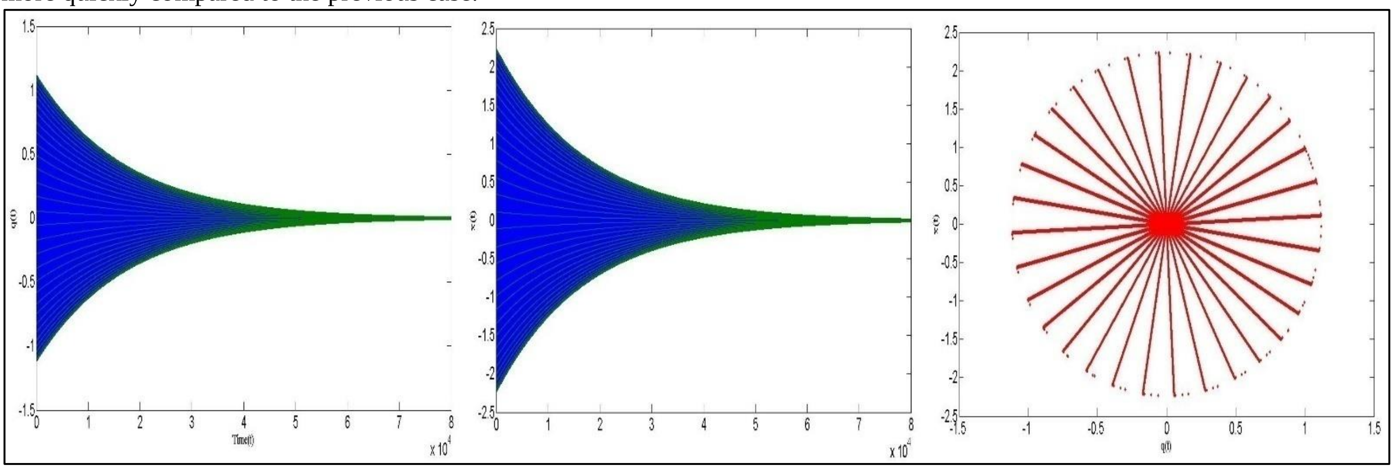

Fig. 2: Un Damped motion of (3) for $\omega_0 = 2$

When the resonant frequency $\omega_0 > 2$, then time taken to approach the equilibrium position is further reduced as illustrated in Figure-3. Thus, in the absence of the damping term, resonant frequency plays a crucial role in attaining stability.

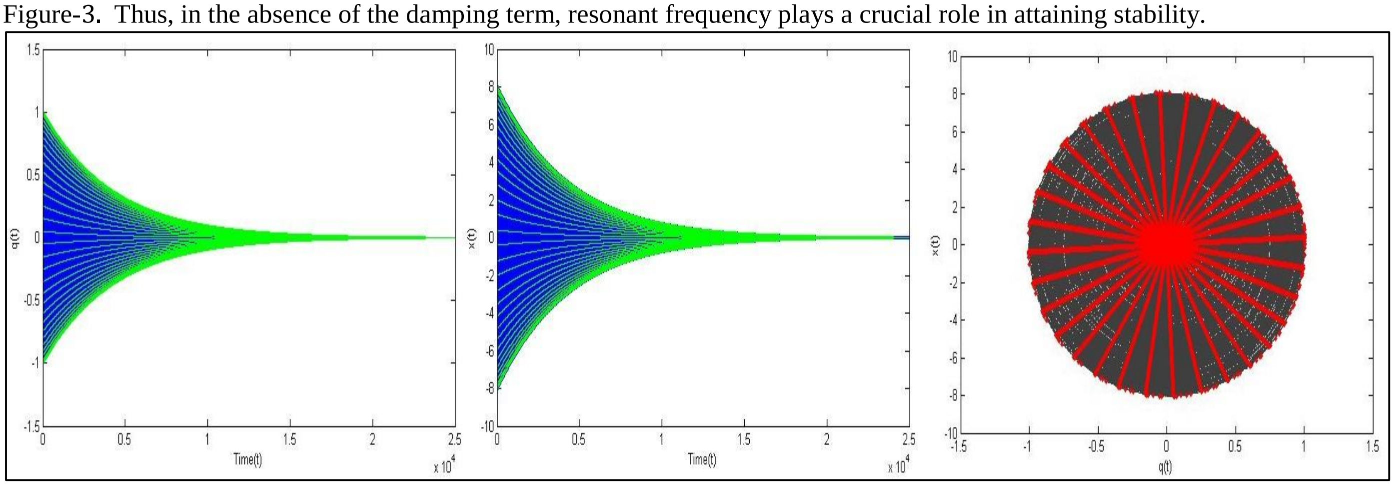

Fig. 3: Un Damped motion of (3) for $\omega_0 = 8$

B. Discrete Fractional Order System

Fractional order form of the system (3) is

$$\frac{d^{\nu}q}{dt^{\nu}} = x(t) \;,\; \frac{d^{\nu}x}{dt^{\nu}} = -\omega_0^2\, q(t) \qquad (4)$$

Where ν is the Fractional order. Equation (4) on discretization becomes

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$$q(t+1) = q(t) + S\,x(t)\ ,\ x(t+1) = x(t) - S\left(\omega_0^2 q(t)\right) \qquad (5)$$

Where $S = \frac{h^{\nu}}{\Gamma(1+\nu)}$. The System (6) has the trivial equilibrium point $(0,0)$.TheJacobian matrix for the system (6) is

$$J(q,x) = \begin{pmatrix} 1 & S \\ -S\omega_0^2 & 1 \end{pmatrix}$$

The eigenvalues $\lambda_{1,2} = 1 \pm i\,\omega_0$ are clearly complex, conjugate of the form $a + i$ bwitha > 0.

The system (5) oscillates for the value of resonant frequency greater than 2 and the range of oscillation diverges with increase in time which is clear indicator that the equilibrium points of the system (5) are unstable. Also, positive real part in the Eigenvalues represents spiral moving outwards, the phase portrait is provided to strengthen the result.

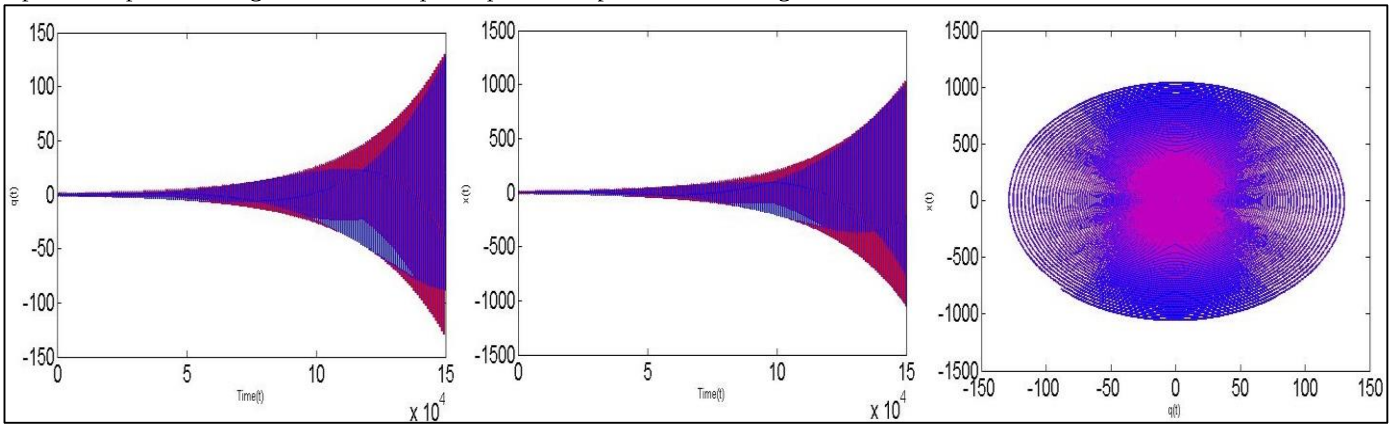

Fig. 4: Un Damped motion of (5) for $\omega_0 = 8$

The oscillations of the system (5) takes a long time for its oscillating range to vary. Figure-5 explains that the system forms a limit cycle shown by phase portrait.

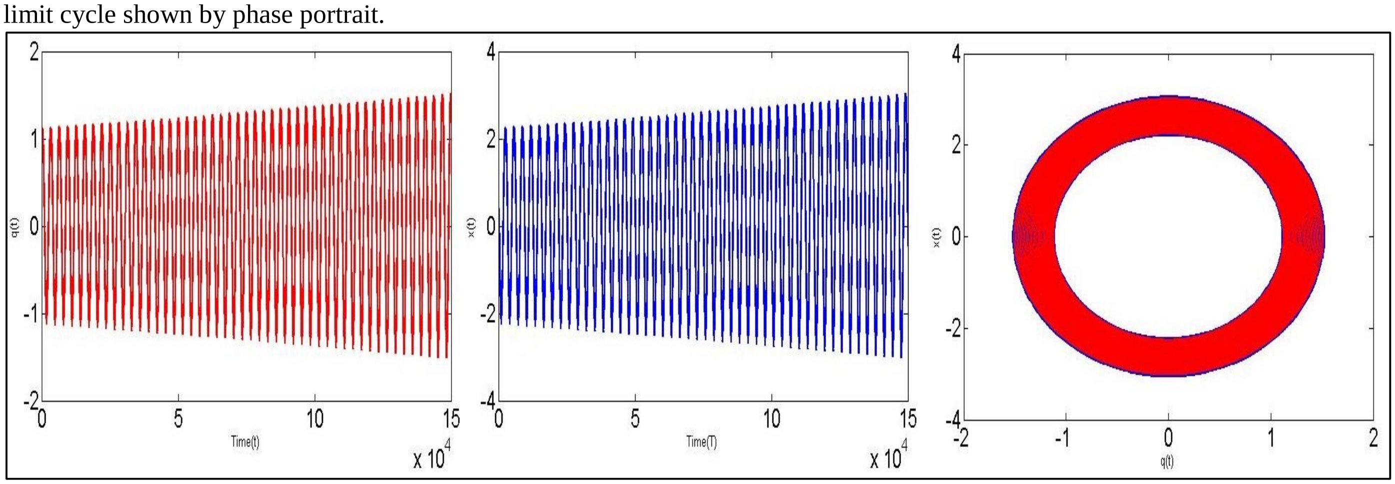

Fig. 5: Un Damped motion of (5) for $\omega_0 = 2$

When the value of ω_0 is reduced to less than 2. The range of the Oscillations of the system remain the same for long time thus the system ends up in forming a limit cycle. The Figure-6 illustrates the uniform oscillation of the system for $\omega_0 = 0.4$ and limit cycle formed in phase portrait.

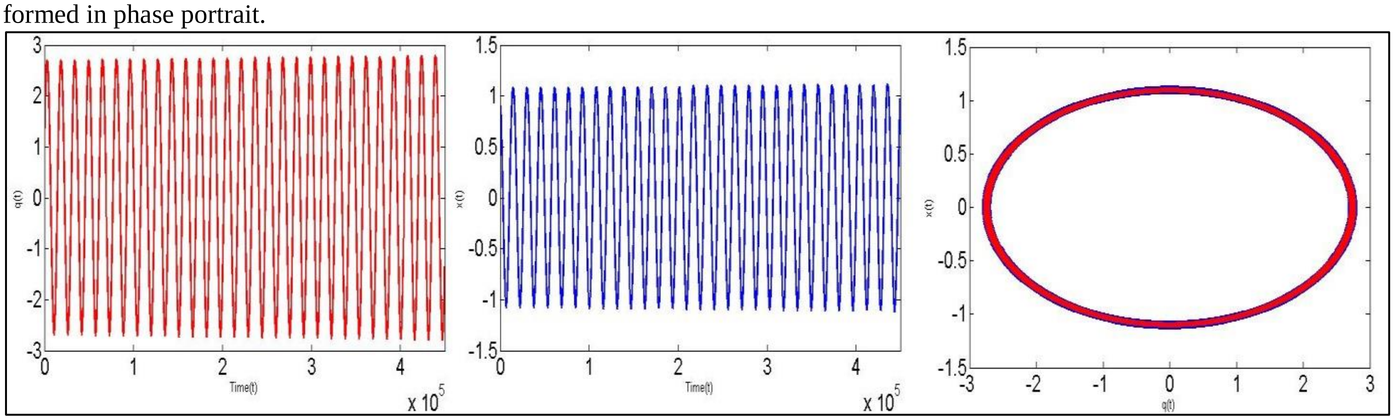

Fig. 6: Un Damped motion of (5) for $\omega_0 = 0.4$

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III. CONCLUSION

Stability and oscillatory properties of both continuous and Discrete Fractional 2-D models describing undamped series RLC circuit are analyzed by linearization technique. Time plots and phase portraits exhibit the dynamics of the Series RLC circuit. The effect of the value of resonant frequency on the stability is visually presented.

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