GRD Journal for Engineering | Volume 2 | Issue 12 | November 2017
ISSN: 2455-5703

The Effect of Uneven Tension on a Surface of a Polymer and New Method of Determining the Line of Highest Tension

Krishna Poddar
Department of Engineering Maths
Chinmaya International Residential School

Abstract

In this research I ventured to find a relation between the eccentricity of an elliptical droplet with the force it experiences on a polymer. All model of finding the force on a polymer assumed the force will be distributed equally across the polymer. However, in reality such idealistic situations are rare. Thus through this exploration a new method could be developed to first find the areas which experienced the highest force and the highest tension and thus are susceptible to break, and to find the mechanics of the water droplet and model the uneven tension as a vector diagram. I conducted experiments to conclude that there is indeed a relation with the eccentricity of a water droplet with the force it experience and also that the major axis of the ellipse points towards the line of highest tension. These results show a very successful attempt to model such phenomenon. My results concluded that the total force can be determined with an accuracy of 2%. My final result was F=e±2% which sows the obvious and very strong relation between the force experienced by the polymer and its effect on the water droplet and how these uneven forces are able to attract the droplet in a manner which depicted the force itself. Through this exploration, a new method of finding the mid-point of the ellipse with a general equation of $Ax^2+Bxy+Cy^2+Dx+Ey+F=0$ was also discovered which indeed presented substantial results and aided the whole process as it deemed to be a simpler and easy to calculate.

Keywords- Relation between the Eccentricity of an Elliptical Droplet with the Force it Experiences on a Polymer

I. INTRODUCTION

Strain and stress are very important factors when building and structure whether it be at the scale of a spacecraft or the size of pacemaker. Weak point in any material when subjected to long term tension or high levels of tension are the most susceptible to damage and have a high probability of degenerating or deforming from that point.

In this study a new method has been developed to indicate the line of highest tension on a plane of a polymer which is under uneven forces thus producing uneven tension throughout the plane. In this method the polymer remains intact and is a completely mathematical model of finding the line of highest tension which can further be analyzed to know the magnitude of the force that is acting on the line and prevent the areas near the highest tension to be weak.

This method will make it easier for civil engineers to determine the weak areas of any material which can then be strengthened in order to make the material able to withstand the high pressure it will experience.

An experiment was conducted to prove and support the findings and aid in the making of the mathematical modelling of the function which is able to determine the magnitude of the force that is present at the line of highest tension.

II. METHODOLOGY AND EXPERIMENTATION

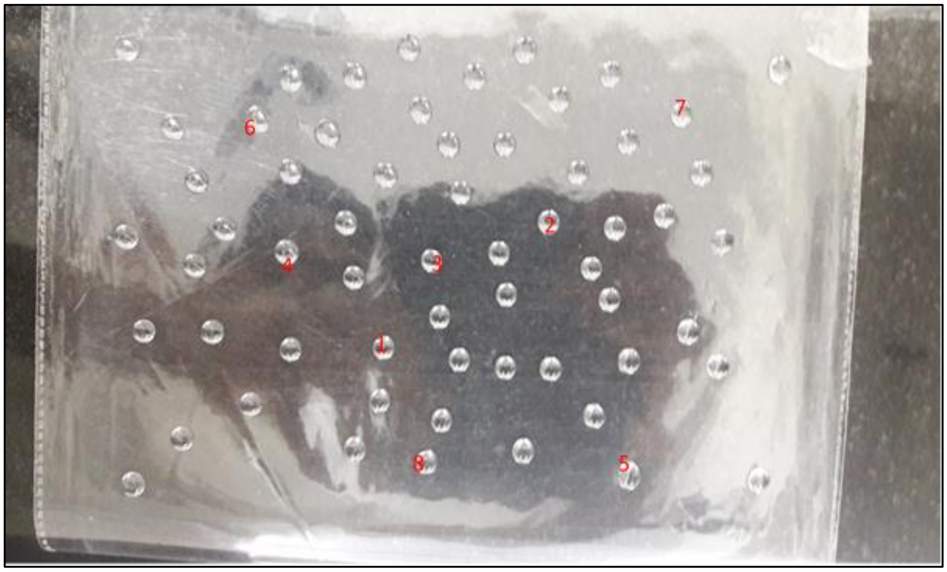
In this method of finding the line of highest tension, droplets of glycerol are used to determine their elliptical shape and build a model around the eccentricity of the ellipse and the magnitude of the force at the line of the highest tension and also determine the precise position of the line of highest tension by measuring the major axis of the elliptical droplets of glycerol.

A. *Procedure*

- Prepare a sheet of polymer and attach it to a surface
- Subject it is uneven forces by hanging a known mass of 50g (0.5N) at the edge of the sheet
- Place approximately same volume of droplets of glycerol at random locations on the sheet.
- Immediately take a photography of the set up in order to analyze it using the software 'GeoGebra 5'
- Make sure to avoid parallax error and take the photography perpendicular to the sheet of polymer
- Import the images in the software

This procedure was followed for the experimentation and provides an accurate depiction of the position and size of the droplet which allows the software to analyze the image to present an accurate representation of the ellipse that forms and formulate the equation of the ellipse. Further analysis of the equations of the ellipses such as finding the equation of the major axis of each ellipse and the average of all indicated the line of highest tension on the polymer. Furthermore, the eccentricity of the ellipse has a direct relationship with the magnitude of the force on the line of highest tension.

B. Experimental Data



This layer of polymer was under a force of 0.5N by hanging 50g of mass from the edge of the sheet. This caused the sheet to be under uneven tension at different locations as the components of the force were not distributed equally throughout the sheet.

After the droplets of glycerol were gently place at random position on the polymer, immediately a photograph was taken in order to preserve the original shape of the droplet before the surface tension broke and the droplet lost its shape. The droplets that were chosen to be analyzed are numbered at shown in the photograph above.

After processing the image through the software, equation of all the ellipses could be formulated by plotting 5 point on the circumpherence of the droplet in the photograph and get an accurate resemblance of the elliptical shape that the droplet forms.

Droplet number	*Equation of the droplet*
1	$-1.06419x^2 + 0.05828x\,y - 0.85455y^2 + 0.66045x + 2.13097y = 1.45014$
2	$-1.25829x^2 + 0.07562x\,y - 0.9617y^2 + 3.28959x + 3.73398y = 5.96756$
3	$-1.1295x^2 - 0.20419x\,y - 0.98788y^2 + 1.80805x + 3.62899y = 3.78653$
4	$-1.37928x^2 - 0.12395x\,y - 1.26401y^2 - 0.48507x + 4.57275y = 4.21945$
5	$-1.9214x^2 + 0.00991x\,y - 1.15778y^2 + 7.15382x + 1.16951y = 6.95394$
6	$-1.45357x^2 - 0.27622x\,y - 1.26079y^2 - 0.5524x + 6.42493y = 8.45189$
7	$-1.19x^2 + 0.04x\,y - 0.85y^2 + 5.11x + 4.38y = 11.36$
8	$-1.38156x^2 + 0.13424x\,y - 1.01068y^2 + 1.60328x + 1.11557y = 0.81221$

These are in the form of

$$Ax^2+Bxy+Cy^2+Dx+Ey+F=0$$

This form of equation will be used hence forward while refereeing to the coefficient of the variable in tables.

C. Reasoning

The equations obtained indicate the shape of the droplets to be elliptical to be in nature when under uneven tension. Droplets when under no net force should be perfectly hemispherical in order as the surface tension of the droplet will form a shape to maximize the surface area and minimize the volume and the hemisphere is the optimal shape for it to achieve such requirements.

However, when the droplet experiences a net force, the molecules closer to the line of highest tension are pulled by the unbalanced forces. The hydrogen bonds and the vander forces of attraction cause the molecules of the droplet to stay together and remain a droplet. This causes distortion of the shape of the droplet stretching it from the side closest to the line of highest tension.

Thus the major axis of the ellipse can be used in order to determine the line of highest tension as it will be pointing to the areas which are experiencing higher tension and force. Thus, the equations of the ellipses that were obtained were further analyzed to determine the line of the major axis for all the droplets under observation and hence find a mathematical model in order to find the line of highest tension.

D. Data Analysis

First, the coordinates of the center of the ellipse should be determined in order to find the length of the major axis.
Using the formula:

$$X_c = \frac{2CD - BE}{B^2 - 4AC} \qquad Y_c = \frac{2AE - BD}{B^2 - 4AC}$$

A	B	C	D	E	F	Xc Final	Yc final
-1.06	0.06	-0.85	0.66	2.13	-1.45	0.347128097	1.265192756
-1.26	0.08	-0.96	3.29	3.73	-5.97	1.369039735	1.999751656
-1.13	-0.2	-0.99	1.81	3.63	-3.79	0.644403355	1.768242085
-1.38	-0.12	-1.26	-0.49	4.57	-4.22	-0.256915629	1.825726141
-1.92	0	-1.16	7.15	1.17	-6.95	1.861979167	0.504310345
-1.45	-0.28	-1.26	-0.55	6.42	-8.45	-0.440356313	2.596547527
-1.19	0.04	-0.85	5.11	4.38	-11.36	2.191227376	2.628028879
-1.38	0.13	-1.01	1.6	1.12	-0.81	0.607667812	0.59356278

Xc- X coordinate for the center of the ellipse
Yc - Y coordinate for the center of the ellipse
From the coordinates of the center of the ellipse we could proceed to find the length and the Cartesian equation of the line of major axis in order to find the line of highest tension.
However, the 'xy' term in the general equation shows a peculiar characteristic of the ellipse. This shows that the ellipse has been rotated by a function of Θ from the x axis in the positive direction (anti-clockwise)
The equation of the function Θ is:

$$\Theta = \arctan \frac{C - A - \sqrt{(A-C)^2 + B^2}}{B} \quad \text{for } B \neq 0$$

Using this equation, the rotational function of the ellipses was determined

A	B	C	D	E	F	Θ (rad)
-1.06	0.06	-0.85	0.66	2.13	-1.45	-0.13914983
-1.26	0.08	-0.96	3.29	3.73	-5.97	-0.130301196
-1.13	-0.2	-0.99	1.81	3.63	-3.79	0.480035181
-1.38	-0.12	-1.26	-0.49	4.57	-4.22	0.392699082
-1.92	0	-1.16	7.15	1.17	-6.95	0
-1.45	-0.28	-1.26	-0.55	6.42	-8.45	0.487298593
-1.19	0.04	-0.85	5.11	4.38	-11.36	-0.058554372
-1.38	0.13	-1.01	1.6	1.12	-0.81	-0.168939094

This angle shows the deviation of the point of apogee and perigee of the rotated ellipse when compared to a normal ellipse.
To find the major axis, a parallel line from the center of the ellipse should be taken and from the point on the circumference of the ellipse the angle of rotation of the ellipse should give the point of apogee of the ellipse.

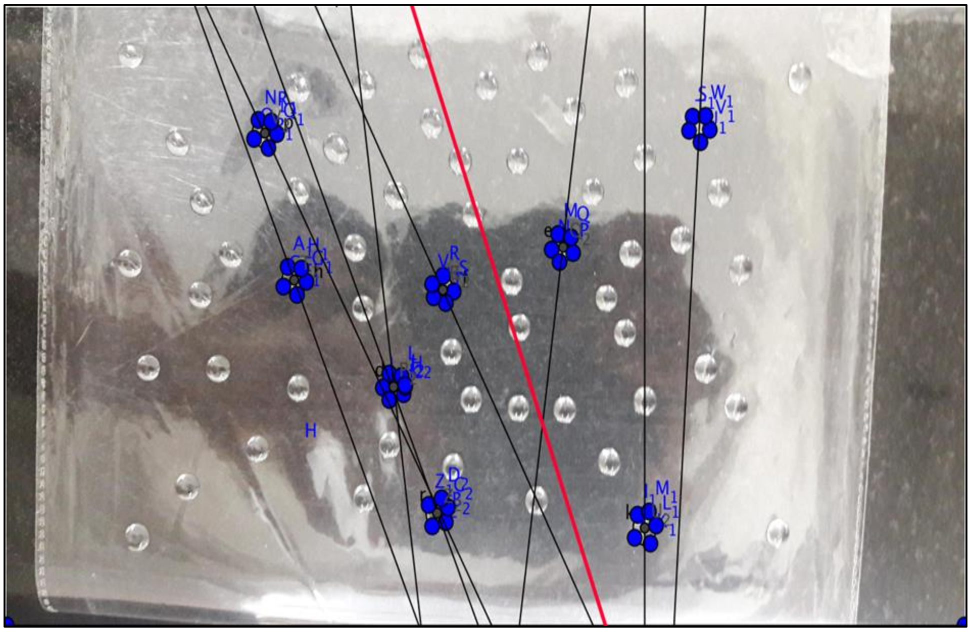
A line adjoining the point of apogee and the center of the ellipse found before will give the line of the major axis of the ellipse. A line perpendicular to the major axis at the center of the ellipse will give the equation of the minor axis which will be useful to find the magnitude of the force which is later discussed in this paper.
The following equations are given in the formation of

$$Ax + By = C$$

Ax	By	C
0.06859	0.02841	0.03408
0.06332	0.02623	0.0541
-0.0662	-0.00855	-0.03358
0.05959	0.03066	0.09244
0.06426	0.03405	0.06033
0.06762	-0.00886	0.07476
0.06494	0.00381	0.13247

The average of each component of the equation gives us the equation

$$0.03697x + 0.01325y = 0.06008$$



In the diagram above the black lines indicate the equation of the major axis of all the ellipses that were considered and the red line to show the line of highest tension that was achieved.

III. THE LINE OF HIGHEST TENSION (LOHT)

With the objective to find the line of highest tension, the mathematical model of proves that the line of the major axis of the ellipse has a directional vector towards the line of highest tension. It is also proven by the fact that the droplets closer to the line of highest tension has their line for the major axis to have a lower gradient pointing directly towards the red line with the point of the intersection of the line of major axis and the line of highest tension not being distinctly far apart. As the droplets go further away from the line of highest tension, the gradient of the major axis comes closer to the gradient of the line of highest tension making it almost parallel. This simply states that the droplets closer to the line of highest tension are pointing directly to the line whereas the major axis of the droplets that are away from the line show a less evident direction towards the line. This supports the theory that the force of attracting by the forces decreases as the distance between the droplets increase.

A. Analysis

To prove that the major axis of all the droplets are pointing towards the line of highest tension and the directional vector of the major axis tends towards the directional vector of the line of highest tension as the droplet goes further away from the line, we can consider the perpendicular distance of the center of the drop with the line, the angle and which the line of major axis intersects with the derived line and the distance from the center of the drop to the pint of intersection of the major axis and the line of highest tension.

We do this in order to determine and confirm the theory as the hypothesis and observations suggest that as the drop will increase in distance, the major axis and the LOHT will become almost parallel and thus at the point of their intersection will far from the droplet and also the angle will be very small.

B. Data

In the table below is the data required acquired from the graph and the following equations to calculate the values:

C. Distance of the center of the droplet from LOHT:

- Draw a perpendicular line from the center of the droplet to the LOHT
- Equate the equation of LOHT and the equation of the perpendicular line to find the point of intersection (x, y)
- a•b = 0
- the dot product of the directional vectors will be zero
- Retrieve the coordinates of the center of the ellipse calculated above (xc, yc)
- Use the distance formula:

$$\sqrt[2]{(y-y_c)^2+(x-x_c)^2}$$

D. Angle between the LOHT and the major axis at the point of intersection

- Calculated from the graphing software

- Use the coordinates of the center and the point of intersection and any point on the line.
- Calculate the angle in the clock-wise direction to be positive
- The three value of angles are negative because they were on the different side of the LOHT causing the angle measured on the software to be from the LOFT to the line of major axis.
- Measure angle from the major axis to the LOHT

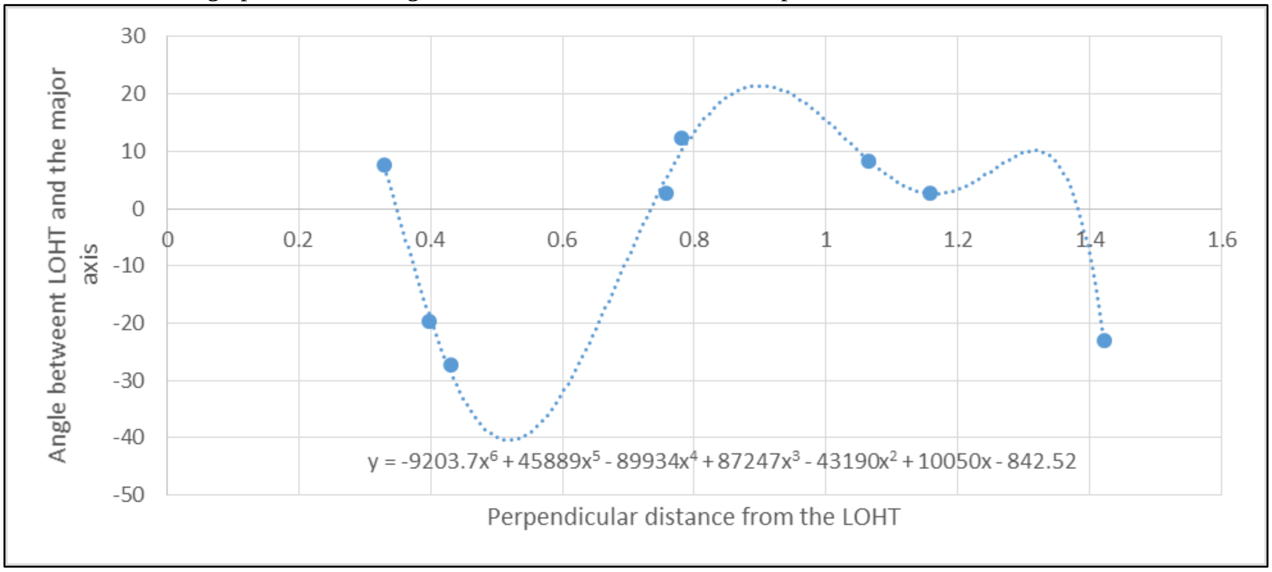
E. Distance between the center of the droplet and the point of intersection between the major axis and LOHT

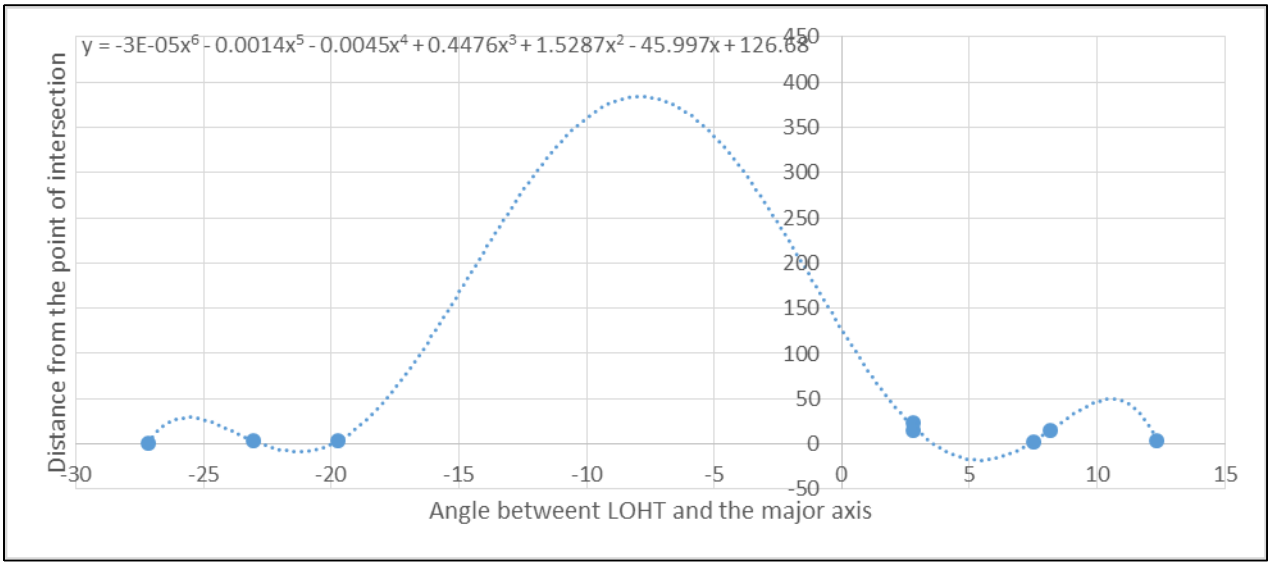
- Equate the equation of LOHT and the equation of the perpendicular line to find the point of intersection (x, y)
- Retrieve the coordinates of the center of the ellipse calculated above (xc, yc)
- Use the distance formula:

$$\sqrt[2]{(y-y_c)^2+(x-x_c)^2}$$

Distance from the LOHT	*Degrees*	*Distance centre from intersection*
0.78055	*12.36099147*	*3.64618*
0.43036	*-27.19314991*	*0.94172*
0.32955	*7.504028243*	*2.5235*
1.15772	*2.778272349*	*23.88652*
0.39698	*-19.72120731*	*3.62713*
1.06489	*8.196734217*	*15.59989*
1.42168	*-23.07644816*	*3.62713*
0.75662	*2.779991222*	*15.59989*

There can be two useful graphs that can be generated from the calculated data present above:





The dotted line indicates the trend lines that can be generated for the given graphs.

F. Data Processing

From the equation of graph 2, the angle at which the maximum distance from the point that can be achieved can be calculated by the set of equation:

$$\frac{dy}{dx} = -3E-05x^6 - 0.0014x^5 - 0.0045x^4 + 0.4476x^3 + 1.5287x^2 - 45.997x + 126.68 \quad = 0$$

Solving the equations 3 values of x are obtained (10.77, 5.39,-7.83)
Taking the second derivative if the equation:

$$\frac{d^2y}{dx^2} - 3E-05x^6 - 0.0014x^5 - 0.0045x^4 + 0.4476x^3 + 1.5287x^2 - 45.997x + 126.68$$

and substituting the obtained values of x we can determine and prove that at x= -7.83 and x= 10.77 there exists a maxima as the value of the second derivative is negative.
Using this value back in the equation the maximum can be calculated to be = 383.07 for
x = -7.83
This proves that distance of the intersection point between the LOHT and the major axis will be maximum when the angle at which they intersect is -7.83º
Using the measure of the angle for the maximum distance of the intersection, a line of the equation
y = -7.83
can be plotted on graph 1 in order to find the points in which the distance from intersection point was maximum and the perpendicular distance from the center of the droplet to the LOHT will also be the greatest.
In order to calculate that, we first obtain the points of intersection of the line
y = -7.83 with the line $y = -9203.7x^6 + 45889x^5 - 89934x^4 + 87247x^3 - 43190x^2 + 10050x - 842.52$ by equating them and finding the possible value of x.
The points of intersections found to be are (0.37, -7.83), (0.70, -7.83) and (1.4, -7.83)

The $\frac{dy}{dx}$ $-9203.7x^6 + 45889x^5 - 89934x^4 + 87247x^3 - 43190x^2 + 10050x - 842.52$

at the given values of x = [0.37,0.7,1.4] will give the gradient of the tangent to show the rate of change in the perpendicular distance of the drop from the LOHT and the angle between the major axis and the LOHT.
In order to find the maximum perpendicular distance, we need to consider the maxima of the first derivate as that will provide the angle at which the maximum perpendicular distance is present. For this we will need to equate the second derivate of the equation to 0.

Thus $\frac{dy}{dx}$ $-9203.7x^6 + 45889x^5 - 89934x^4 + 87247x^3 - 43190x^2 + 10050x - 842.52$
=

$$y = -55222.2x^5 + 229445x^4 - 359736x^3 + 261741x^2 - 86380x + 10050$$

This equation shows the gradient of the equation at all possible values of x. To find the maximum gradient we will calculate the maxima of the first derivative of the equation for which we will differentiate it again to make the second derivate of the equation.

$\frac{d^2y}{dx^2}$ $-9203.7x^6 + 45889x^5 - 89934x^4 + 87247x^3 - 43190x^2 + 10050x - 842.52$
=
$y = -276111x^4 + 917780x^3 - 1079208x^2 + 523482x - 86380$
After equating it to 0 the following values of x are obtained
x= [1.2549, 1.0305, 0.6858, 0.3528]
These values of x show the maxima and minima in the gradient of the equation. To obtain only the maxima for these values we can differentiate it again and do the sign test to confirm the maximas
The x values of the maximas were found to be
x = [1.2549, 1.0305, 0.6858]
as the value of the equation was negative after substituting these x values
The global maxima can be found by substituting the x value of x in the equation and finding the greatest value of y. The coordinates of the global maxima is
(1.2549 , -0.3923)
Thus this shows that greatest perpendicular distance with respect to angle.
Plugging the obtained values of x into the original equation we can obtain the value of angle that gives the maximum perpendicular distance
$-9203.7x^6 + 45889x^5 - 89934x^4 + 87247x^3 - 43190x^2 + 10050x - 842.52$ at x= 1.2549
y = 7.65º
This is very close to the absolute value of angle that gives the maximum distance to the point of intersection of the major axis to the LOHT which was 7.83º. We consider the absolute value because distance cannot be negative thus the calculated values were the magnitude of the measured angle.

The difference is negligible thus we can conclude that as the perpendicular distance increases the distance of the point of intersection also increases as both the equations are strictly increasing. The calculations show that the angle at which the maximum distance of intersection point was found is similar to the maximum perpendicular distance of the droplet from the LOHT. This proves the relationship between the perpendicular distance and the distance of intersection of the major axis and LOHT and directly proportional as the maximum of both cases happen at similar angles.

This proves that the droplets close to the LOHT has a significantly experience more force and point directly towards the line. This is the situation where the perpendicular distance is very small so the distance of the intersection point will also be small. This shows that the major axis will be 'pointing' directly at the LOHT. On the other hand, the drops far from the LOHT show less 'pointing' towards the line. This is where the perpendicular distance of the droplet is larage thus the distance taken for the intersection of the major axis with the LOHT is also large. Thus the major axis is pointing faintly to the LOHT. This shows that the orientation of the droplets is not random but is influenced by the LOHT.

G. Calculating the Magnitude of the Force Applied

Eccentricity is the deviation of the curve of an ellipse from circularity. The eccentricity of the droplet will show its relative position when compared to the line of highest tension. As the closer the droplet gets it will get attracted more, the eccentricity of the ellipse will be high. However, by taking the average of many droplets we can approximate the force that the polymer is under as there is a direct correlation between the eccentricity of the elliptical shape and the force that was applied on the polymer. In this section, a mathematical model has been formulated to calculate the force that the polymer was under.

The first eccentricity of the ellipse is the ratio of the distance of the focus from the center to the length of the semi major axis. Another way to express is in terms of the length of the semi major axis and semi minor axis.

The general formula is:

$$\sqrt{1-\frac{b^2}{a^2}}$$

a: length of the semi major axis

b: length of the semi minor axis

However, the formula for eccentricity of rotated ellipses is as follows:

$$e=\sqrt{\frac{2\sqrt{(A-C)^2+B^2}}{\eta(A+C)+\sqrt{(A-C)^2+B^2}}}$$

Where η is the coefficient to make the root real and is based on the determinant where

$$\begin{bmatrix} A & B/2 & D/2 \\ B/2 & C & E/2 \\ D/2 & E/2 & F \end{bmatrix}$$

η= -1 if the value of the determinant is positive

η= 1 if the value of the determinant is negative

This formula considers the rotation of the ellipse in the Cartesian plane which allows it to find the length of the major axis and minor axis that are not parallel to the 'x' or 'y' plane.

The table shows the calculated values for the ellipses considered:

A	*B*	*C*	*D*	*E*	*F*	*determinant*	***Eccentricity***
-1.06	*0.06*	*-0.85*	*0.66*	*2.13*	*-1.45*	*0.0107855*	***0.453020266***
-1.26	*0.08*	*-0.96*	*3.29*	*3.73*	*-5.97*	*0.0140215*	***0.495373194***
-1.13	*-0.2*	*-0.99*	*1.81*	*3.63*	*-3.79*	*0.002821*	***0.454454893***
-1.38	*-0.12*	*-1.26*	*-0.49*	*4.57*	*-4.22*	*0.025557*	***0.347562337***
-1.92	*0*	*-1.16*	*7.15*	*1.17*	*-6.95*	*0.003557*	***0.62915287***
-1.45	*-0.28*	*-1.26*	*-0.55*	*6.42*	*-8.45*	*0.0108725*	***0.47117466***
-1.19	*0.04*	*-0.85*	*5.11*	*4.38*	*-11.36*	*-0.00609775*	***0.536098494***
-1.38	*0.13*	*-1.01*	*1.6*	*1.12*	*-0.81*	*0.01185225*	***0.530960217***

The average of all the eccentricity give a value of

Average eccentricity = 0.489724616

The total load that the polymer was under was 0.5N
This shows that the eccentricity is very close to the force that the polymer was under with an error of

$$\frac{0.489724616-0.5}{0.5} \times 100 = -2.0550768\% \approx -2.06\ \%\ \text{error}$$

This error is under the acceptable experimental range and produces accurate results.

H. Reasoning

The reason the eccentricity is the mathematical model to determine the force the polymer is under is because the ratio of the length of the focus with the semi major axis shows the lateral stretch the droplet underwent in order to become an ellipse. As the force of the polymer will cause the droplet's major axis to point towards the LOHT, as the force increases the deviation from the circular shape will increase causing a larger eccentricity of the droplet. Thus the average of numerous eccentricity is able to determine the force the polymer is under, as the magnitude of the force is directly proportional to the eccentricity of the ellipse.

IV. CONCLUSION

The above mentioned mathematical model can be used to find the total force a polymer is under and show the line of highest tension. The model final expression can be derived in the following equation:

$$F \propto e$$
$$F = ke$$

Where k is the constant of proportionality which is equal to 1
As the experiment suggests the equation with the errors is finally be

$$F = e \pm 2\%$$

V. EVALUATION

The experiment which could be used to derive many results had its own limitations which might have caused the results obtained to be skewed.
Firstly, due to computational restrains numerous data points were not considered. This could have led to droplets near the vicinity of the LOHT to be considered rather than on the whole sheet. Even though the droplets were spread out on a big area and covered most the sheet, the average of more data points would have given a more accurate result. Thus there a scope of further research by computing more data sets and obtaining a precise models.

Furthermore, the photograph of the polymer with the sheets was taken by a camera by hand. To reduce the parallax error due to the angle camera, it was kept at approximately 90º to the sheet. However, human error in small difference in the angle could have caused skewed results.

Moreover, the graphing software 'GeoGebra' only allowed the conics to be plotted by 5 points on the droplet. The limited computational power restricted this experiment to use an approximate equation on the ellipse generated. By plotting more points, a better outline of the droplet could be found which would provide accurate results.

The time lag between the droplets placed on the sheet and the photograph taken could have affected the extent to which the droplet was affected by the uneven tension of the polymer. Multiple photographs superimposed on different droplets, taken immediately after the droplet was placed would have created the image to the the situation right after the droplets were in contact with the polymer. This would eliminate the errors of loss of shape or the time lag.

BIBLIOGRAPHY

[1] Thomas, George B.; Finney, Ross L. (1979), Calculus and Analytic Geometry (fifth ed.), Addison-Wesley, p. 434. ISBN 0-201-07540-7
[2] Ayoub, Ayoub B., "The eccentricity of a conic section", The College Mathematics Journal 34(2), March 2003, 116-121.
[3] "Classification of Linear PDEs in Two Independent Variables".
[4] S.Chekanov, J.Proudfoot, "Searches for TeV-scale particles at the LHC using jet shapes", Phys. Rev. D 81:114038, (2010). arXiv:1002.3982
[5] http://www.mathwarehouse.com/ellipse/eccentricity-of-ellipse.php
[6] https://math.stackexchange.com/questions/264877/finding-eccentricity-from-the-rotating-ellipse-formula
[7] https://math.stackexchange.com/questions/280937/finding-the-angle-of-rotation-of-an-ellipse-from-its-general-equation-and-the-ot
[8] https://math.stackexchange.com/questions/1102960/finding-eccentricity-directrix-foci-of-diagonal-ellipse-by-rotating-it
[9] http://mathworld.wolfram.com/Ellipse.html
[10] http://www.stewartcalculus.com/data/ESSENTIAL%20CALCULUS%20Early%20Transcendentals/upfiles/topics/ess_at_13_ra_stu.pdf

