

# 2D Contour Design and Analysis of the Divergent Portion of a Satellite Thruster Nozzle using MOC

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## Abstract

In this paper, we are going to design a supersonic nozzle using Method of Characteristics. Minimum length of the supersonic nozzle is found out using this method. Here in this we are focusing on the diverging section of the nozzle and the coordinates are found out using MATLAB program. This program is developed for two dimensional, steady, inviscid flow.

**Keywords-** Design, Supersonic, C-D Nozzle, Minimum Length, Method of Characteristics

## I. INTRODUCTION

The conditions of a two dimensional, steady, isentropic, irrotational flow expressed mathematically by nonlinear differential equation. The method of characteristics is a mathematical method which is used to find solutions for mentioned velocity potential, therefore satisfying given boundary conditions for which the governing partial differential equations (PDEs) become ordinary differential equations (ODEs). Traditionally the supersonic nozzle is divided into two parts. The upstream conditions of the sonic line is not dependent on the supersonic portion. The subsonic portion can be study independently. The sonic flow at the throat can be used latter. We design a type of nozzle giving a parallel and uniform flow at the exit section. It is named by Minimum Length Nozzle with centered expansion, which gives the minimal length compared to the other existing types

### A. Minimum Length Nozzle

In order to expand a flow through a duct from subsonic to supersonic speed convergent - divergent in shape is more preferable in the duct. If the nozzle contour is not proper, chances for shock waves to occur inside the duct is high. The method of characteristics provides a technique for designing the contour of a supersonic nozzle for shock free, isentropic flow, taking into account the multidimensional flow inside the duct. The purpose of this section is to illustrate such an application.

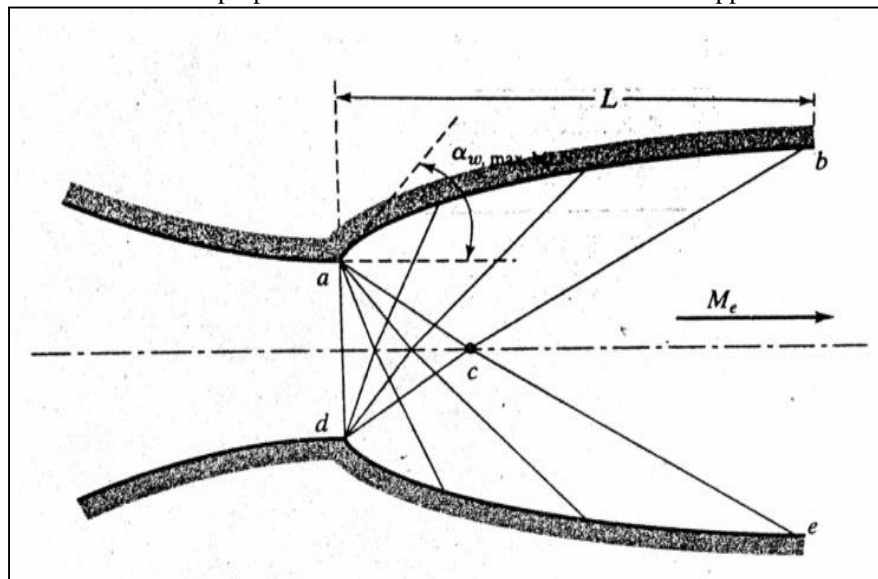


Fig. 1: Straight Sonic Line Minimum Length Supersonic Nozzle

Rocket nozzles are very short in order to minimize weight. Also, in cases where rapid expansions are desirable, such as the non-equilibrium flow in modern gas dynamic lasers, the nozzle length is as short as possible. In such minimum-length nozzles,

the expansion section is shrunk to a point, and the expansion takes place through a centred Prandtl- Meyer wave emanating from a sharp-corner throat with an angle  $\theta W_{max}$ ,  $M_L$  as sketched. The length of the supersonic nozzle, denoted as  $L$  in Fig 1.4 is the minimum value consistent with shock free, isentropic flow. If the contour is made shorter than  $L$ , shocks will develop inside the nozzle.

A fluid element moving along a streamline is constantly accelerated while passing through these multiple reflected waves. For the minimum-length nozzle, the expansion contour is a sharp corner at point  $a$ . There are no multiple reflections and a fluid element encounters only two systems of waves—the right-running waves emanating from point  $a$  and the left-running waves emanating from point  $d$ . Let  $vM$  be the Prandtl-Meyer function associated with the design exit Mach number. Hence, along the  $C^+$  characteristic „cb“,  $v = vM = v_c = v_b$ .

### B. Method of Characteristics

Method of characteristics is a numerical procedure to solve certain classes of flow problems. This procedure enables solutions to be obtained for two dimensional, planar and axisymmetric, and three-dimensional supersonic flows. The procedure may also be employed for all one-dimensional unsteady flows. Method of characteristics has been used for many years to solve problems in various fields of study that are described by hyperbolic partial differential equations. With velocity (magnitude and direction) and Mach number known at points  $A$  and  $B$  and  $v(M)$  are known, so the Riemann invariants  $C_1$  and  $C_{11}$  can be calculated from the equations. Characteristic I from point  $A$  intersects with Characteristic II from point  $B$  at point  $P$ , as shown in the figure. Since  $v + \alpha = \text{constant}$  along a characteristic of Type I and  $v - \alpha = \text{constant}$  along a characteristic of Type II, it follows that, at point  $P$

$$C_I = v_A + \alpha_A = v_P + \alpha_P$$

$$C_{II} = v_B - \alpha_B = v_P - \alpha_P$$

So, we have two equations with two unknowns,  $v_P$  and  $\alpha_P$ , which may be readily solved to obtain

$$v_P = \frac{C_1 + C_2}{2}$$

$$\alpha_P = \frac{C_1 - C_2}{2}$$

Since, from

$$v_P = \left\{ \sqrt{\frac{\gamma+1}{\gamma-1}} \tan^{-1} \left[ \sqrt{\frac{\gamma-1}{\gamma+1}} (M_P^2 - 1) \right] - \tan^{-1} \left( \sqrt{M_P^2 - 1} \right) \right\} \quad (2.9)$$

$v = v(M)$ , the Mach number at  $P$  can be determined. Moreover, the direction of the velocity at  $P$  is known from equation. For isentropic flow, the Mach number can be used to determine the values of the stagnation-to-static-temperature and pressure ratios at point  $P$ . Since  $T_0$  and  $P_0$  are constants for this flow, the static temperature and pressure at  $P$  can be calculated. By working outward from the initial value curve  $LM$ , it is then possible to build a net or mesh, with points connected by the characteristics.

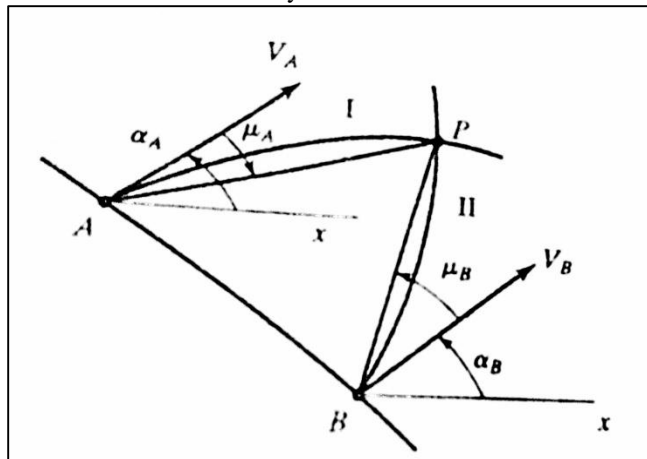
Of course, to complete the process, we need to determine the direction of the characteristics. Since Mach number varies throughout the flow field, the characteristic lines  $AP$  and  $BP$  are not necessarily straight, but rather may be curved. However, if the size of the mesh is kept small enough, the lines  $AP$  and  $BP$  can be approximated by straight lines inclined at the Mach angle to the flow direction. Naturally, the accuracy of this procedure depends on the mesh size. The smaller the mesh, the more accurate the solution is.

To find the direction of the characteristics, we employ

$$\left( \frac{dy}{dx} \right)_1 = \tan(\alpha - \mu)$$

$$\left( \frac{dy}{dx} \right)_2 = \tan(\alpha + \mu)$$

As we have seen, these equations tell us that at a given point the characteristics of family I are inclined to the  $x$ -axis at an angle of  $\alpha - \mu$ , whereas at a given point the characteristics of family II are inclined to the  $x$ -axis at an angle of  $\alpha + \mu$



To solve the forgoing pair of equations, we may use the explicit Euler method that was used to solve ordinary differential equations. However, this situation is somewhat different because we must solve the pair of equations simultaneously. Thus, instead of selecting a set value of the difference  $\Delta x$ , here we write the finite-difference versions of the equations as follows.

$$\left(\frac{dy}{dx}\right)_1 = \frac{y_P - y_A}{x_P - x_A} = \tan(\alpha - \mu)_A = m_1$$

$$\left(\frac{dy}{dx}\right)_2 = \frac{y_P - y_B}{x_P - x_B} = \tan(\alpha - \mu)_B = m_2$$

Because characteristics can be curved, there are other ways to evaluate the right hand side of equations (. For example, we could base the inclination angle on the average value between the initial value line and the target point. This approach leads to an improved calculation of the location of the target point. Applying this method to equations yields

$$\left(\frac{dy}{dx}\right)_1 = \frac{y_P - y_A}{x_P - x_A} = \tan\left[\frac{(\alpha - \mu)_A + (\alpha - \mu)_P}{2}\right] = m_1$$

$$\left(\frac{dy}{dx}\right)_2 = \frac{y_P - y_B}{x_P - x_B} = \tan\left[\frac{(\alpha - \mu)_B + (\alpha - \mu)_P}{2}\right] = m_2$$

These variations will result in a slightly different characteristic and will cause the target point P to appear at a different location from that shown in figure

$$y_P = y_A + m_1(x_P - x_A)$$

$$y_P = y_B + m_2(x_P - x_B)$$

Solving this pair of equations simultaneously yields  $x_P$ , the x location of point P.

$$x_P = \frac{y_A - y_B + m_2x_B - m_1x_A}{m_2 - m_1}$$

## II. MATLAB

The design procedure using the MOC is little bit complex. So the manual calculation is much difficult and time consuming, So by writing this procedure in programming languages, the properties of the grid points can be easily find out even for a unskilled person. This is done using MATLAB.

## III. DESIGN

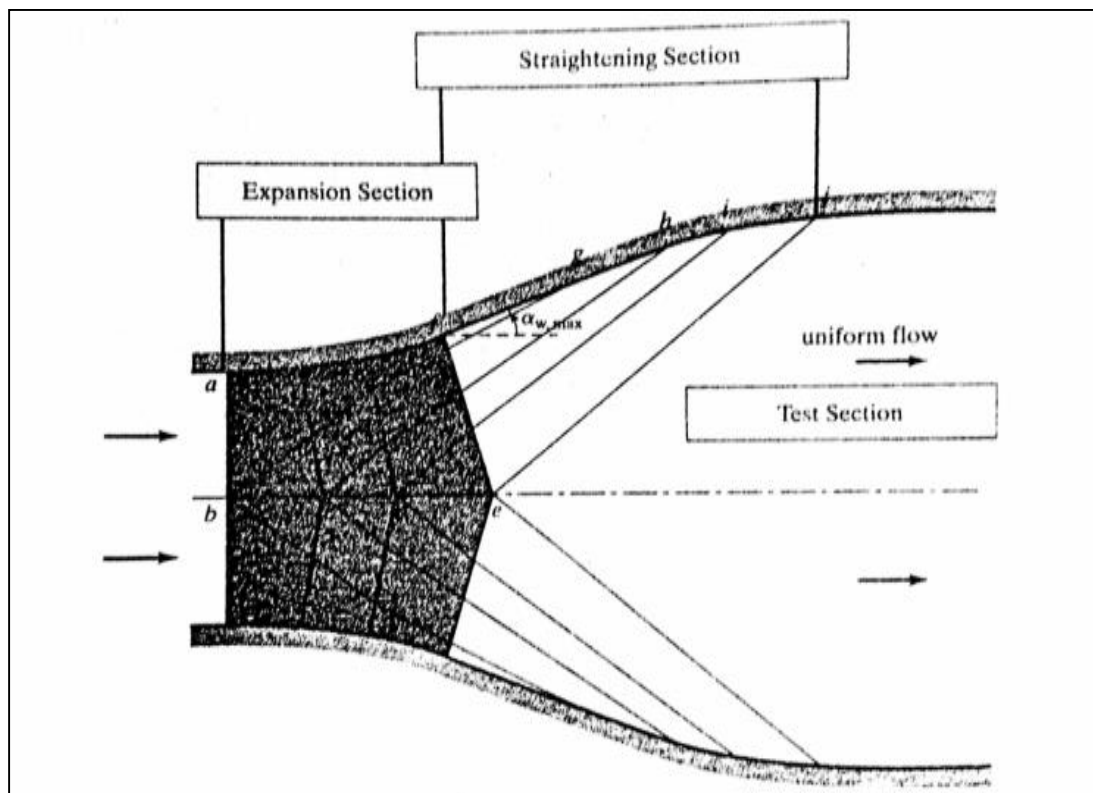
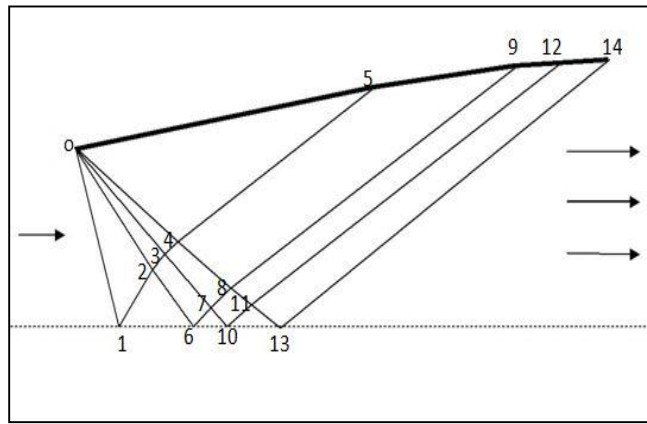


Fig. 2: Wave Pattern in a Supersonic Nozzle Containing Both Expansion and Straightening Section



For  $M_e = 1.8$  and  $\gamma=1.2$ , the prandtl-Meyer spreadsheet solver is used to obtain  $v(M_e) = 24.30252^\circ$ . Therefore,  $\alpha_{max,MLN} = 12.1512^\circ$ . It is assumed that the flow angle at a point 1 is  $0.1525^\circ$ . Each succeeding wave in the expansion fan is assumed to turn the flow by  $4^\circ$ .

The computations at grid points 2,3, and 4 all follow the unit-process expression of Equations. 2.12 And 2.13. The same procedure is used to determine information at grid point 7,8 and 11.

To assume that there is no deflection from the boundary, the flow angle at point 5 is set equal to that at point 4. Likewise  $\alpha_9 = \alpha_8$ ,  $\alpha_{12} = \alpha_{11}$  and  $\alpha_{14} = \alpha_{13} = 0^\circ$ . Because the boundary with known flow angle is intercepted by a type 2 characteristics, we use equation  $v-\alpha=\text{constant}=C_2$  to compute  $v_5, v_9, v_{12}$  and  $v_{14}=24.30252^\circ$ .

The flow angle at point 6, 10 and 13 is  $0^\circ$ , and since each of these point is intercepted by type 1 characteristics, we use equation  $v+\alpha=\text{constant}=C_1$  to determine the corresponding  $v$  values.

Finally, the coordinates of all grid points are computed using equations. To provide more accurate slopes the values of slopes is averaged between points on connecting characteristics. The slop of points on the boundary of nozzle are also averaged. Note that the expansion is assumed to be centered at the location (0, 1).

All the characteristics and properties of every grid points on the nozzle contour are find out. But the aim of this project is to calculate the grid point properties and coordinates of the nozzle contour in an easy way. The required data may be only the output Mach number and the value of  $\gamma$ . So any person with or without knowledge of the nozzle design can design the nozzle within the fraction of seconds. This is done by writing a code in known programming language ie MATLAB.

From the MALAB programming we getting coordinates for the supersonic nozzle. With these points a profile is generated in AUTOCAD software.

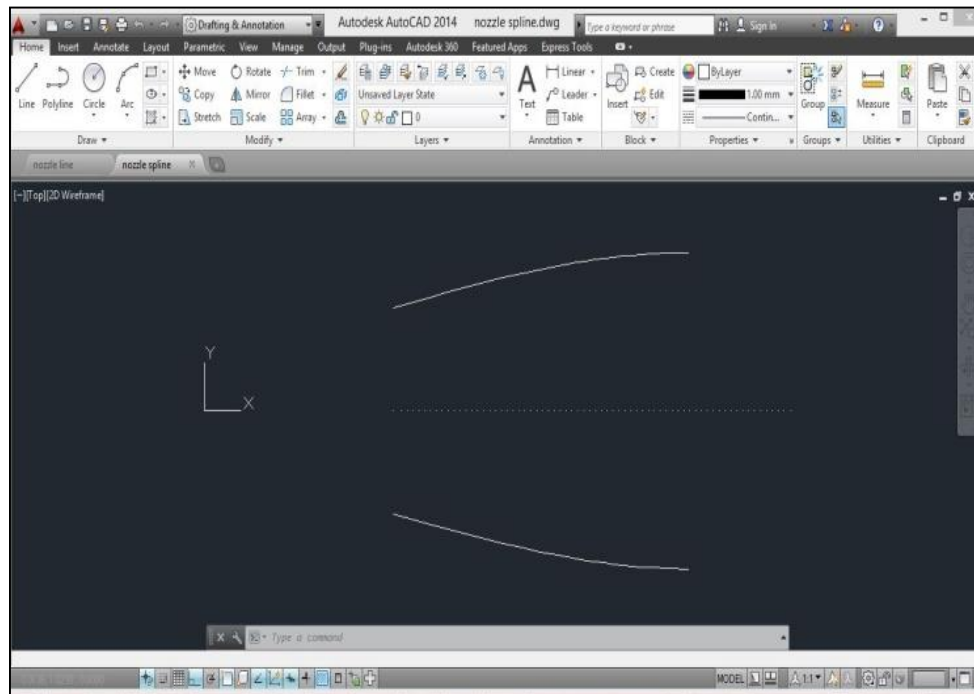


Fig. 3: CAD Drawing of Nozzle Contour

After obtaining the profile the analysis can be done using ANSYS software

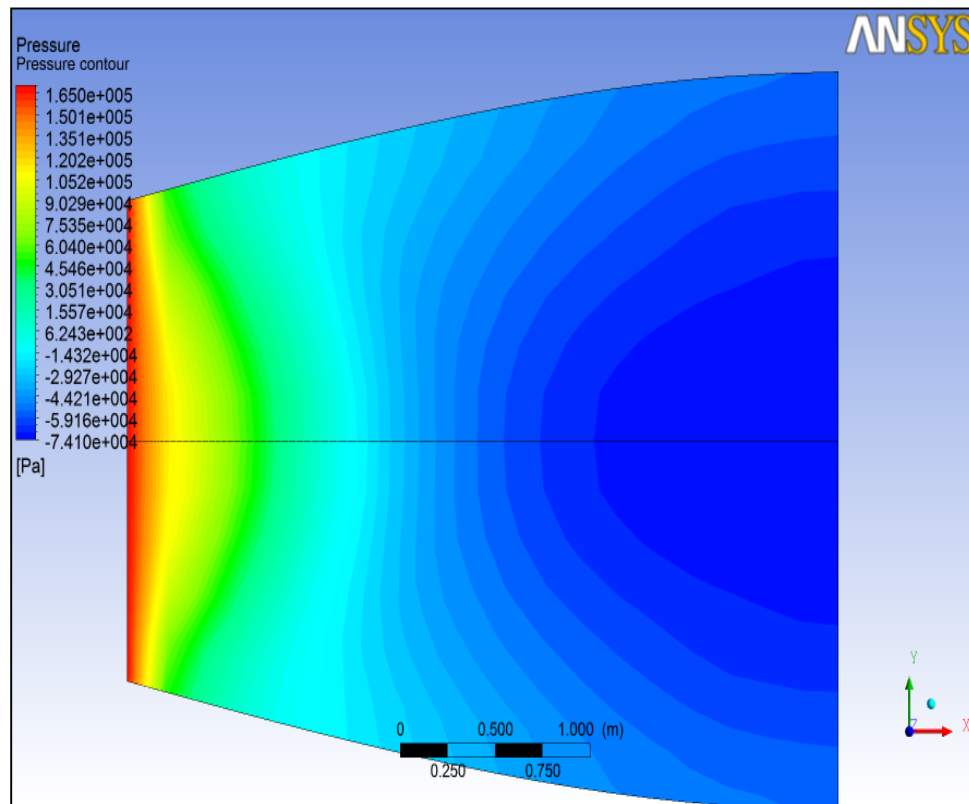


Fig. 4: Pressure Contour

Pressure change across the nozzle is obtained. Along the length of nozzle, pressure is decreasing from inlet to exit. Red colour indicating the maximum pressure and is decreasing at the exit which is indicated by blue colour. That means when the area increases, the pressure decreases, which is the property of nozzle.

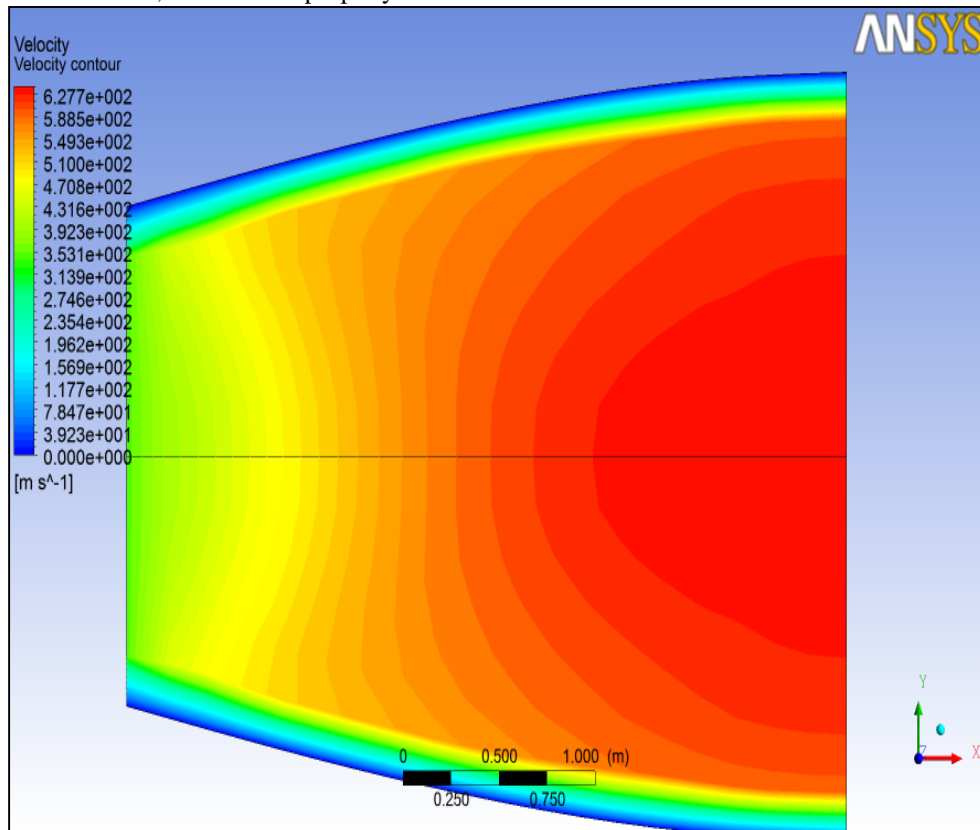


Fig. 5: Velocity Contour

Velocity variation along the nozzle is obtained as shown in fig. velocity is increasing when the flow goes from throat to exit. Along the boundary, there is a boundary layer formation .so near the boundary the velocity is very less which can be shown in result

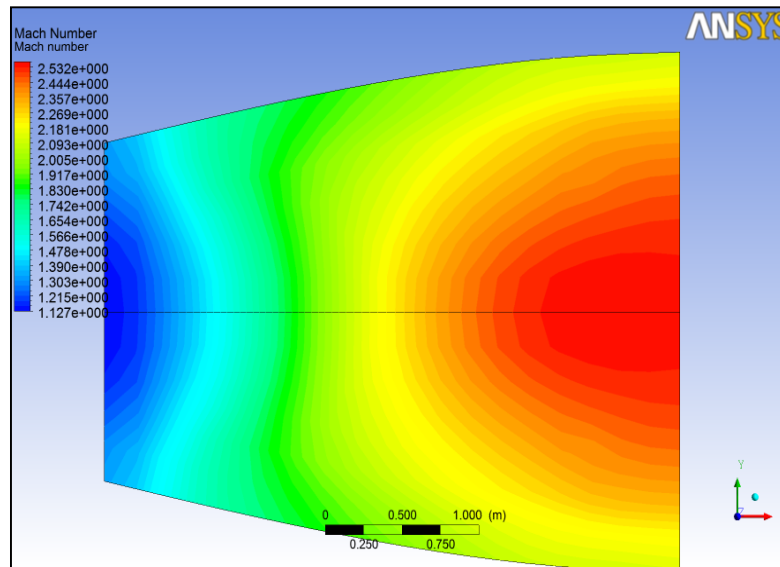


Fig. 6: Mach number Contour

The Mach number variation is the important plot. The Mach number at the throat is given as sonic condition. That is the Mach number near the throat is one, which is increasing along the nozzle. At the exit the Mach number is expected as 1.8. The obtained result shows some variation from the expected output. This may be due to the slight errors in nozzle contour. These errors may due to the lesser number of waves are considered.

#### IV. CONCLUSIONS

Study over the method of characteristics successfully done. There are mainly two procedures for the calculation of nozzle properties. One is point to point method and other is region to region method. The selected procedure is point to point method. Because it simpler than the second one.

The procedure of method of characteristics, application software is prepared, which is done in java. This gives the characteristics and coordinates of grid point for the given data. The input data is output Mach number and values of  $\gamma$ . Here the properties of coordinates are generated for the output Mach number 1.8 and  $\gamma$  is 1.2.

The result from application software gives almost similar values that obtained from manual calculations. Using the obtained coordinates, the cad drawing of nozzle contour is prepared. The nozzle contour was a perfect diverging section.

The obtained nozzle contour ready to check, whether obtained design is correct or not, which needs the CFD analysis. This is done in ANSYS fluent. The proper boundary condition is given and some good results of pressure, velocity, Mach number contour and corresponding plots are obtained. The Mach number variation is the more important parameter. The output Mach number is expected as 1.8. But the result show some variation from the predicted value. For actual design of nozzle, it may consider thousands of waves and corresponding grid points. When the number of coordinates increases, the contour will be more accurate. But here to avoid complexity and for the easiness, it is considered as four waves. This results lesser number of coordinates for nozzle contour. Then the nozzle contour will not be perfect. It may have some errors that is the expected reason behind the variation of analyzed value and actual value.

The application software is the main part of our project. With this software any person can designed nozzle within seconds and can analyses it. Even for the unskilled person or person with less knowledge about method of characteristic can easily designed the nozzle.

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